

Pomeron exchange and exclusive electroproduction of ρ -mesons in QCD

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Abstract

A Pomeron-exchange model of exclusive electroproduction of ρ -mesons is examined using a dressed-quark propagator. It is shown that by representing the photon- ρ -meson-Pomeron coupling by a nonperturbative, confined-quark loop, one obtains predictions for ρ -meson electroproduction that are in good agreement with experiment.

Keywords: Pomeron; ρ -meson Electroproduction; Dyson-Schwinger Equations; Confinement; Nonperturbative QCD Phenomenology.

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1. Introduction. It is observed that high energy hadron elastic cross sections are forward peaked and rise slowly with energy \sqrt{s} . This is attributed to the exchange of a phenomenological object known as the Pomeron. Attempts to describe the Pomeron as the exchange of two nonperturbatively-dressed gluons between quarks typically treat the quarks as free (i.e. all quark lines are represented by free, constituent-quark propagators). However, it has been argued that quark confinement is necessary to obtain a reasonable description of the Pomeron in terms of a gluon exchange [1]. In this work we investigate the role of confined-quark loops in a Pomeron-exchange model.

We examine a model, a modification of that in Ref. [2], in which the Pomeron mediates the long range interaction between a confined quark and a nucleon. Our model provides a framework in which some nonperturbative aspects of Pomeron exchange can be investigated. We apply the model to, and present predictions for, diffractive ρ -meson electroproduction cross sections and compare them to recent data from the ZEUS Collaboration [3] and previous measurements [4,5] in a wide range of energy and photon momentum squared.

Herein we illustrate that, in ρ -meson electroproduction, agreement with experiment can be obtained by including the full effect of the quark loop, using a finite-width ρ -meson Bethe-Salpeter amplitude. We find that, even at high energies, the inclusion of a confined-quark loop has considerable impact on such exclusive processes.

2. A model for Pomeron exchange. In the model of Ref. [2], the Pomeron-nucleon coupling is described by the vertex $F_\mu(t) \equiv 3\beta_0\gamma_\mu f(t)$, where $-t$ is the Pomeron momentum squared, $f(t)$ is a phenomenologically determined form factor, β_0 is a coupling strength, and the factor of 3 counts the number of quarks in the nucleon. With this vertex, the pp elastic scattering amplitude due to single Pomeron exchange is

$$T_{pp \rightarrow pp} = i\bar{u}(p_3)F_\mu(t)u(p_1)G_{\mathcal{P}}(s,t)\bar{u}(p_4)F_\mu(t)u(p_2), \quad (1)$$

where $G_{\mathcal{P}}(s,t)$ is assumed to have the form

$$G_{\mathcal{P}}(s,t) = (-i\alpha' s)^{\alpha(t)-1}, \quad (2)$$

with $\alpha(t) = 1 + \epsilon + \alpha't$. The parameters ϵ , α' , and β_0 , are fixed by requiring that $T_{pp \rightarrow pp}$ reproduces the observed differential and total pp elastic scattering cross sections. It was shown in Ref. [2] that with $f(t) = F_1(t)$, the isoscalar nucleon electromagnetic form factor, this model is able to describe the large body of pp and $\bar{p}p$ elastic scattering data. In this model the Pomeron couples to hadrons as an isoscalar photon. Throughout this work, following Ref. [6], we use $\epsilon = 0.08$, $\alpha' = 0.25 \text{ GeV}^{-2}$, $\beta_0 = 2.0 \text{ GeV}^2$ and $F_1(t) \equiv (4M_N^2 - 2.8t)/(4M_N^2 - t)/(1 - t/0.7)^2$.

In employing this model in a calculation of the small- x behavior of the proton structure function a form factor $(\mu_0^2/(\mu_0^2 + p^2))$, $\mu_0 = 1.2 \text{ GeV}$ was introduced, for each off-shell quark leg of momentum p that couples to the Pomeron, in order to ensure convergence of the quark triangle diagram used to describe the $\gamma^*\gamma^*P$ vertex [6]. In the application of this model to exclusive ρ -meson electroproduction, the same form factor is employed in the quark triangle diagram that describes the photon- ρ -meson-Pomeron ($\gamma\rho P$) coupling. Using an “on-shell approximation” and a free, constituent-quark propagator with quark mass $M_q = \frac{1}{2}m_\rho$, a value of the loop integral is inferred from the $\rho \rightarrow e^+e^-$ electromagnetic decay constant. In our calculation, good agreement with the electroproduction data is obtained without such a form factor.

3. A model for $\gamma\rho P$ coupling. We employ a field theoretic framework, formulated in Euclidean space¹, in which the impulse approximation to the ρ -meson

¹We use the metric $\delta_{\mu\nu} = \text{diag}(1, 1, 1, 1)$ and Hermitian, traceless, Dirac matrices which satisfy $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$.

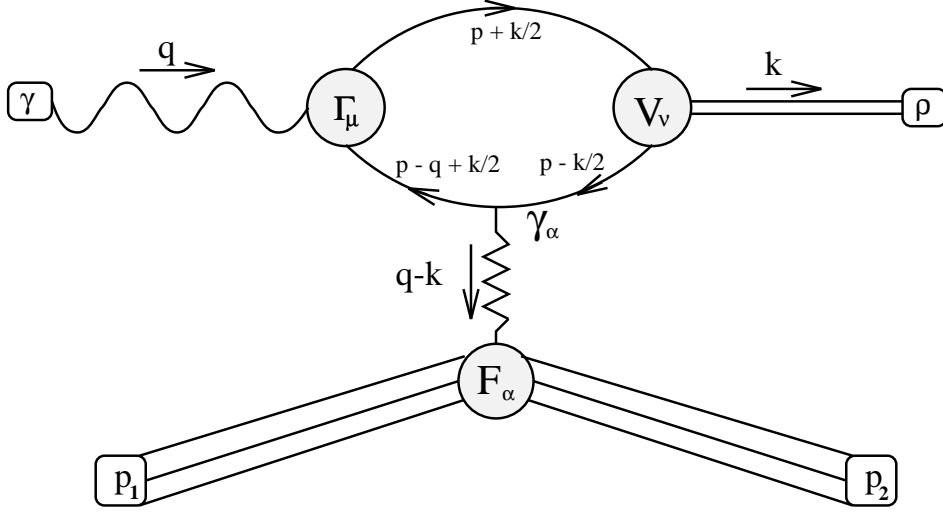


Fig. 1. Lowest order diagram for the ρ -meson electroproduction current given by Eq. (3).

electroproduction current matrix element is

$$\langle p_2 m_2; k \lambda_\rho | J_\mu | p_1 m_1 \rangle = 2\beta_0 t_{\mu\alpha\nu}(q, k) \varepsilon_\nu^{\lambda\rho}(k) G_{\mathcal{P}}(\bar{w}^2, t) \bar{u}(p_2) F_\alpha(t) u(p_1), \quad (3)$$

which is illustrated in Fig. 1. Here $u(p_1)$ and $\bar{u}(p_2)$ are incoming and outgoing nucleon spinors, $\varepsilon_\nu^{\lambda\rho}(k)$ is the polarization vector of the ρ -meson, and $t = -(p_1 - p_2)^2 \leq 0$. The explicit factor of 2 arises because of the equivalence under charge conjugation, C , of the two leading order contributions, in which the Pomeron strikes the quark or antiquark. In this way we are naturally led to a “quark counting rule” for the $\gamma\rho P$ vertex function, $t_{\mu\alpha\nu}(q, k)$. The $\gamma\rho P$ vertex in Eq. (3) is analogous to the nucleon-Pomeron vertex, $F_\alpha(t)$, from pp scattering, in the sense that it represents the Pomeron-hadron coupling, and in our model is given by

$$t_{\mu\alpha\nu}(q, k) = \frac{3e_0}{(2\pi)^4} \text{tr} \int d^4p S(p_+) \Gamma_\mu(p_+, p_+ - q) S(p_+ - q) \gamma_\alpha S(p_-) V_\nu(p_-, p_+), \quad (4)$$

where $p_\pm = p \pm \frac{1}{2}k$. In Eq. (4) $S(p) = [i\gamma \cdot p A(p^2) + B(p^2)]^{-1}$ is the dressed-quark propagator, $\Gamma_\mu(p, p')$ is the dressed photon-quark vertex, and $V_\nu(p, p')$ is the ρ -meson Bethe-Salpeter amplitude, which will be discussed below. The trace is over Dirac indices.

In Fig. 1 the square of the quark-nucleon energy is $w^2 = -(p_1 + q - \frac{1}{2}k - p)^2$, which depends on the loop integration variable p . To make contact with Refs. [6] we use an average value $\bar{w}^2 = -(p_1 + q - \frac{1}{2}k)^2$ which allows one to write the electroproduction amplitude as a product of Pomeron-hadron vertices and Pomeron-exchange amplitude, $G_{\mathcal{P}}(\bar{w}^2, t)$. We employ this “factorization Ansatz” throughout this work. In our framework, the implication of this Ansatz can be explored and will be discussed elsewhere.

As remarked above, herein each of the terms appearing in the hadron current, J_μ in Eq. (3), is taken from Refs. [6] except the $\gamma\rho P$ vertex, $t_{\mu\alpha\nu}(q, k)$, a calculation of which is the focus of this work.

The “on-shell approximation” of Refs. [6] can be reproduced in our approach by taking the ρ -meson BS amplitude to be

$$V_\nu(p - \tfrac{1}{2}k, p + \tfrac{1}{2}k) = (2\pi)^4 \delta^4(p) \frac{m_\rho^4}{N_\rho} \gamma_\nu. \quad (5)$$

Using a bare photon vertex $\Gamma_\mu = \gamma_\mu$, free constituent-quark propagator $S(p) = (i\gamma \cdot p + \tfrac{1}{2}m_\rho)^{-1}$, and the above form of ρ -meson amplitude, the loop integration in Eq. (4) can be performed trivially. The only unknown parameter is the ρ -meson vertex normalization, N_ρ , which is fixed by calculating the $\rho \rightarrow e^+e^-$ decay width, in the same approximation. In Fig. 2 we compare the calculated results obtained in this way with the available data.

In our calculation of the loop integral in Eq. (4) we use the photon-quark vertex, Γ_μ , and quark propagator, S , developed in studies of QCD based on the Dyson-Schwinger equations (DSEs) in Euclidean space. (A review can be found in Ref. [7].) DSE studies have been employed successfully in the study of hadron and electromagnetic interactions of pseudoscalar and vector mesons. In the study of vector mesons the simple Gaussian Ansatz for the ρ -meson BS amplitude

$$V_\nu(p - \tfrac{1}{2}k, p + \tfrac{1}{2}k) = \left(\gamma_\nu + \frac{k_\nu \gamma \cdot k}{m_\rho^2} \right) \frac{e^{-p^2/\Lambda^2}}{N_\rho}, \quad (6)$$

has proven phenomenologically successful [8]. In Eq. (6), k is the center-of-mass momentum of the ρ -meson and p is the quark-antiquark relative momentum. The normalization, N_ρ , is fixed by the canonical Bethe-Salpeter normalization and the range, Λ , is determined by requiring that the model reproduce the $\rho \rightarrow e^+e^-$ decay width.

The essential elements of our calculation of both the $\gamma\rho P$ vertex and $\rho \rightarrow e^+e^-$ width are the quark propagator and photon-quark vertex.

The photon-quark vertex, Γ_μ , has been studied in some detail within the DSE approach [7]. The quark propagator and photon-quark vertex must be dressed in a consistent manner in order to satisfy the Ward-Takahashi identity (WTI). In this work we use the vertex [9],

$$i\Gamma_\mu^{\text{BC}}(p, p') = \frac{i}{2}\gamma_\mu f_1(p^2, p'^2) + (p + p')_\mu [i\gamma \cdot \frac{p+p'}{2} f_2(p^2, p'^2) + f_3(p^2, p'^2)], \quad (7)$$

with $f_1(p^2, p'^2) = A(p^2) + A(p'^2)$, $f_2(p^2, p'^2) = (A(p^2) - A(p'^2))/(p^2 - p'^2)$, and $f_3(p^2, p'^2) = (B(p^2) - B(p'^2))/(p^2 - p'^2)$. This vertex satisfies the WTI and transforms correctly under C , P , T , and Lorentz transformations. It has the

correct perturbative limit and is free of kinematic singularities. However, it is not unique. The addition of transverse terms is important in connection with true gauge covariance and multiplicative renormalizability, but contributes little or not at all to physical observables [10].

The dressed-quark propagator, $S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$:

$$\begin{aligned}\bar{\sigma}_S(x) &= \frac{1-e^{-b_1x}}{b_1x} \frac{1-e^{-b_3x}}{b_3x} (b_0 + b_2 \frac{1-e^{-b_4x}}{b_4x}) + \bar{m} \frac{1-e^{-2(x+\bar{m}^2)}}{x+\bar{m}^2}, \\ \bar{\sigma}_V(x) &= \frac{2(x+\bar{m}^2) - 1 + e^{-2(x+\bar{m}^2)}}{2(x+\bar{m}^2)^2},\end{aligned}\tag{8}$$

with $x = p^2/\lambda^2$, $\bar{\sigma}_S = \lambda\sigma_S$, $\bar{\sigma}_V = \lambda^2\sigma_V$. The qualitative features of this algebraic form follow from numerous studies of quark-DSE using realistic model forms for the gluon propagator and quark-gluon vertex. The parameters are fixed in Ref. [11] by requiring that the model provide a good description of the pion; e.g. f_π , π - π scattering lengths, pion charge radius, and form factor: $b_0 = 0.118$, $b_1 = 2.51$, $b_2 = 0.525$, $b_3 = 0.169$, and $b_4 = 10^{-4}$. In Eq. (8) $m = \lambda\bar{m} = 6.7$ MeV is the bare quark mass and $\lambda = 0.568$ GeV is the momentum scale. This dressed-quark propagator has no Lehmann representation and hence can be interpreted as describing a confined particle since this feature is sufficient to ensure the absence of quark production thresholds in $t_{\mu\alpha\nu}(q, k)$. With this choice of quark propagator, the Ball-Chiu vertex in Eq. (7) and ρ -meson Bethe-Salpeter amplitude in Eq. (6) are fixed and there are *no* free parameters. Using this dressed-quark propagator the $\rho \rightarrow e^+e^-$ decay width is reproduced with $\Lambda = 0.495$ GeV.

4. Results and discussion. We calculate the exclusive ρ -meson electroproduction cross section, which can be separated into a transverse term and a longitudinal term: $d\sigma/d\Omega'_e dE'_e = \Gamma(\sigma_T(q^2, W) + \epsilon\sigma_L(q^2, W))$ where Γ and ϵ are the usual virtual photon flux and photon polarization parameter, respectively [12]. The transverse and longitudinal cross sections depend only on the invariant mass, $W^2 = -(p_2 + k)^2$, and the space-like momentum squared of the virtual photon, q^2 . We define the Hand photon momentum $K_H = (W^2 - M_N^2)/(2M_N)$, and obtain (in the electron scattering plane $\phi = 0$),

$$\begin{aligned}\frac{d\sigma_T}{d\Omega} &= \frac{1}{(2\pi)^2} \frac{M_N}{4W} \frac{|\vec{k}|}{K_H} \frac{1}{2} \sum_{\text{spins}} (J_x J_x^\dagger + J_y J_y^\dagger)_{\phi=0}, \\ \frac{d\sigma_L}{d\Omega} &= \frac{1}{(2\pi)^2} \frac{M_N}{4W} \frac{|\vec{k}|}{K_H} \frac{q^2}{\omega^2} \sum_{\text{spins}} (J_z J_z^\dagger)_{\phi=0}.\end{aligned}\tag{9}$$

Here J_μ is the electromagnetic hadron current defined in Eq. (3), M_N is the nucleon mass and ω is the photon momentum in the center-of-momentum

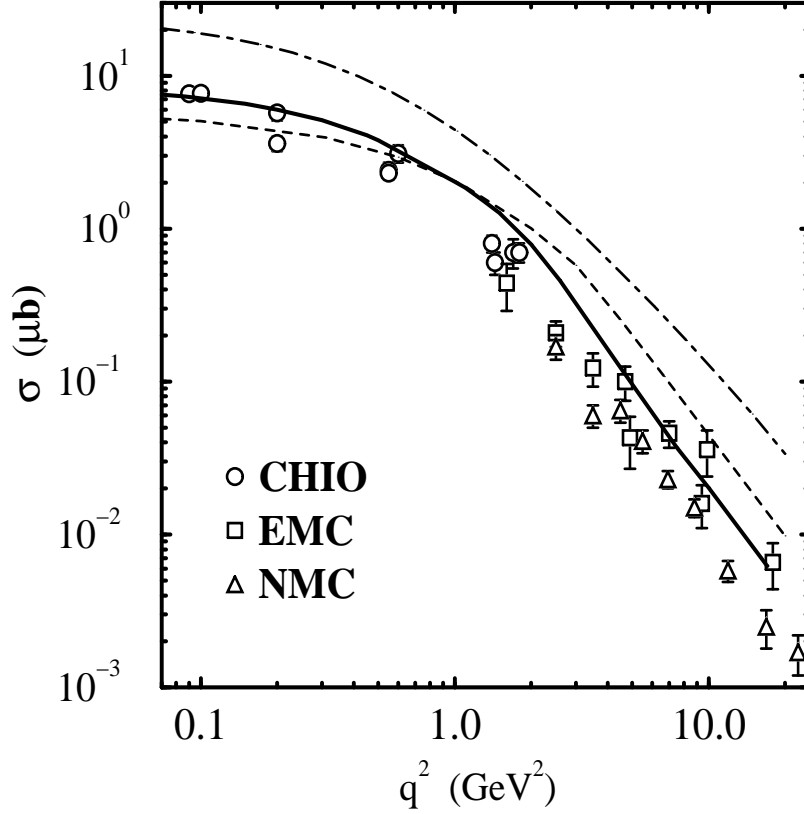


Fig. 2. The total ρ -meson electroproduction cross section. The solid line is our model, the dash-dotted line is obtained using the “on-shell approximation”, and the dashed line is our model with a simplified dressed-quark propagator. The data are from Refs. [4] and error bars shown are statistical. The curves are calculated at $W = 10$ GeV, and $\epsilon = 0.85$ (average value given by EMC in Ref. [4]).

frame.

In Fig. 2 we compare our predictions of the total cross sections for exclusive ρ -meson electroproduction with the data from the CHIO, EMC and NMC Collaborations [4], in the energy region $5.5 < W < 16$ GeV. The result of our model using the ρ -meson amplitude, Eq. (6), the Ball-Chiu vertex, Eq. (7), and confined-quark propagator, Eq. (8) is shown as a solid curve in Fig. 2. It is in good agreement with the data. No parameters were varied to achieve this result. The small- q^2 behavior is tightly constrained and determined by the fact that the quark-photon vertex satisfies the Ward identity [11]. The dash-dotted curve is obtained by employing the “on-shell approximation” (using the δ -function form of ρ -meson amplitude, Eq. (5), and a free, constituent-quark propagator). This is our estimation of the result that would be obtained in the model of Refs. [6] if no quark-Pomeron vertex is introduced. It overestimates the data. We can fit the data if we introduce a quark-Pomeron form factor $\mu_0^2/(\mu_0^2 + p^2)$, with $\mu_0 = 1.2$ GeV. Our calculation suggests that this form factor models the quark substructure of the $\gamma\rho P$ vertex amplitude and that the cross

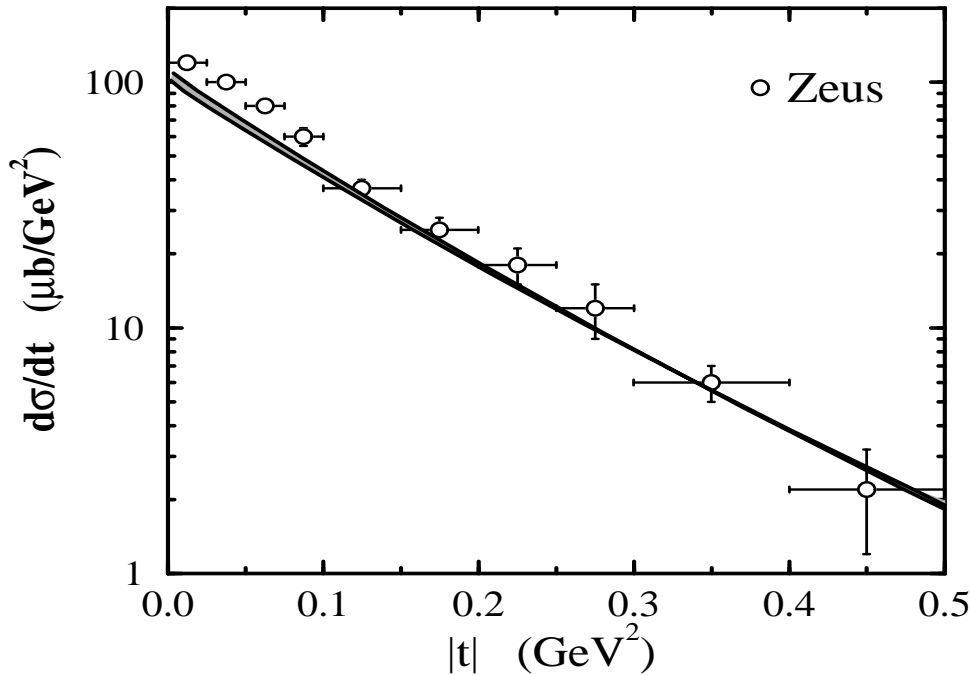


Fig. 3. The differential cross section for exclusive ρ -meson photoproduction ($q^2 = 0$) obtained by the ZEUS Collaboration over a range of energies $60 < W < 80$ GeV [3]. The region between the two solid curves is the prediction of this work for the same range of energies.

section in exclusive processes is sensitive to the Bethe-Salpeter amplitude of the final-state meson. Therefore an analysis of the data using the “on-shell approximation” in the model of Refs. [6] may be misleading.

Our prediction for the differential cross section of photoproduction of ρ -mesons at $60 < W < 80$ GeV (and $q^2 = 0$) is also in good agreement with the recent data from ZEUS [3], as shown in Fig. 3.

The energy range in Fig. 3 is $60 < W < 80$ GeV, while $W = 10$ GeV in Fig. 2. This emphasizes that the Pomeron phenomenology is successful over a large energy domain. This point is illustrated again in Fig. 4, in which, as expected from the form of $G_{\mathcal{P}}(s, t)$, the cross sections rise very slowly with W . The rate of this increase is determined by the value of ϵ in Eq. (2).

In the topmost curve in Fig. 4 ($q^2 = 0$) Pomeron exchange dominates for $W > 6$ GeV. For $W < 6$ GeV the Pomeron contribution alone does not provide for agreement with the data. This is consistent with the expectation that, on this small- W domain, other mechanisms, such as quark or meson exchanges, provide a significant contribution. As an illustration of this point we include the results (dashed curve in Fig. 4) for ρ -meson photoproduction ($q^2 = 0$) calculated using the meson-exchange model of Ref. [13].

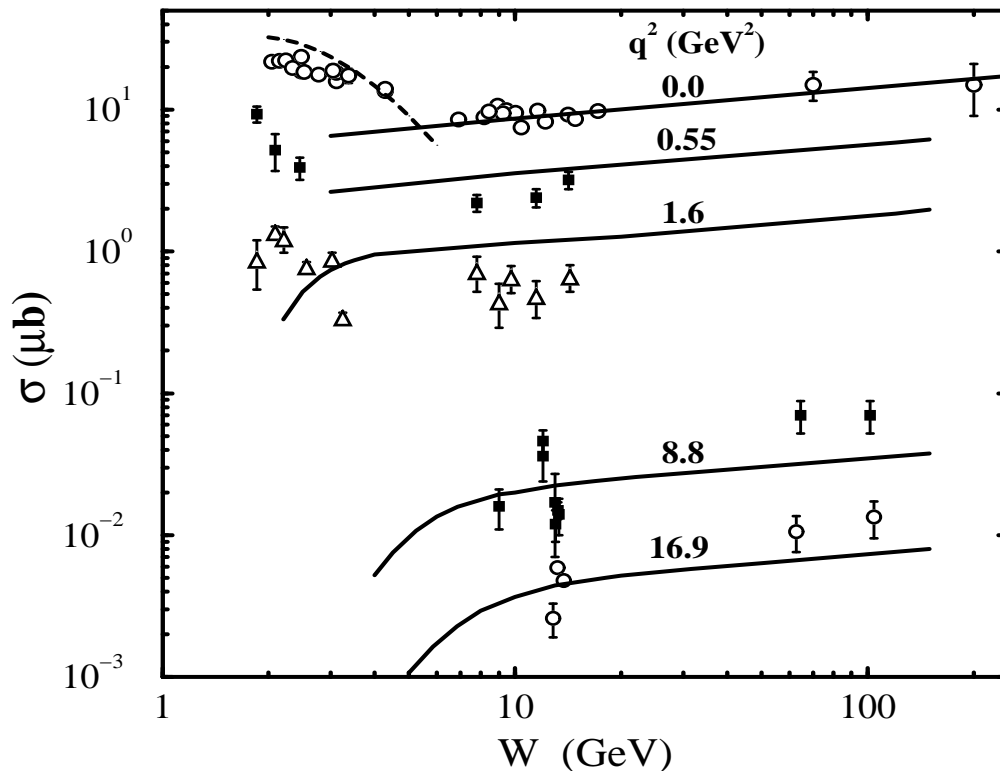


Fig. 4. Energy dependence of the total ρ -meson electroproduction cross section at several values of q^2 . The solid line is this work and the dashed line is a meson exchange calculation [13]. The data shown are from Refs. [3–5]. Error bars shown are statistical.

As discussed above, the photon-quark vertex and ρ -meson amplitude are completely determined once the dressed-quark propagator is specified. To explore the model sensitivity we have also calculated the ρ -meson electroproduction cross section using the simplified dressed-quark propagator: $S(p) = (1 - \exp[-(1 + p^2/M^2)])/(i\gamma \cdot p + M)$, where M is an effective-mass parameter. This model preserves the realization of quark confinement described above but is inadequate for a description of low energy pion properties. With $M = 0.5$ GeV, $\Lambda = 0.465$ GeV reproduces the $\rho \rightarrow e^+e^-$ width and the electroproduction cross section determined in this case is the dashed curve in Fig. 2. The shape and magnitude of this curve are insensitive to M on the range $0.4 < M < 0.7$ GeV. Comparing the solid and dashed curves in Fig. 2, we find that our conclusion concerning the importance of the loop integration does not depend on the exact form of quark propagator employed. Quantitative agreement with data requires the form of Eq. (8).

5. Conclusion. We have calculated ρ -meson electroproduction cross sections with the photon- ρ -Pomeron coupling described by a confined-quark loop with a finite-width ρ -meson Bethe-Salpeter amplitude. Good agreement with the available data is obtained without employing a form factor at the quark-

Pomeron vertex. This does not entail, however, that the Pomeron is pointlike because information about its non-pointlike nature may be contained in the parameterization of the Pomeron-exchange amplitude, $G_{\mathcal{P}}(s, t)$.

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